

## Lab 2 Notes: Higher order Functions

### ① Higher order Functions & currying

- Functions can return functions!

```
def outer():  
  def inner():  
    return "something"  
  return inner
```

```
>>> outer()  
Function  
>>> outer()()  
inner  
"something"
```

- currying: use higher order functions to convert function that takes multiple args into a chain of functions that each take a single arg

```
def curried-add(x):  
  def helper(y):  
    return x+y  
  return helper
```

```
>>> curried-add(2)(5)  
7  
      f1  
      f2
```

- Env diagram:

Global  
curried-add L → func curried-add(x) [P=G]

f1: curried-add [P=G]

x L

~~curried-add~~

helper L → func helper(y) [P=f1]

PV L

f2: helper [P=f1]

y L

PV L

## ② Lambda Expressions

• Lambda expr evaluate to functions. They specify:

- 1) Parameters
- 2) Return expr

• Syntax:  $\text{lambda } \langle \text{parameters} \rangle : \langle \text{return expr} \rangle$

EX:  $\text{lambda } x : x$   
A function that takes  $x$  and returns  $x$

• Properties of lambda <sup>expr</sup> functions:

- An expression that evaluates to a value. Evaluating the lambda expr does NOT create/modify variables
- Creates an anonymous lambda function with no intrinsic name
- Note that we can assign a variable to be equal to a lambda expression

$f = \text{lambda } x, y : (x + y) * 2$

$\ggg f(3, 5)$   
16

$\ggg (\text{lambda } x, y : (x + y))(3, 5)$   
16

ENV. Diagram Example:

```

1 def compose (f, g):
2   return lambda x: f(g(x))
3
4 f = compose (lambda x: x*x,
5             lambda y: y+1)
6 result = f(12)
    
```

Annotations in the code:  
 - Line 2:  $f$  is annotated with  $r.v. f4$  and  $r.v. f3$ .  
 - Line 4: The entire  $f = \dots$  expression is annotated with  $r.v. f1$ .  
 - Line 6:  $f(12)$  is annotated with  $r.v. f2$ .

Global

compose L  $\longrightarrow$  func compose (f, g) [P = G]

f L  $\xrightarrow{f1}$  result L  $\xrightarrow{f2}$  169

f1: compose [P = G]

f L  $\longrightarrow$  func  $\lambda(x)$  <line 4> [P = G]

g L  $\longrightarrow$  func  $\lambda(y)$  <line 5> [P = G]

PV L  $\longrightarrow$  func  $\lambda(x)$  <line 2> [P = f1]

f2:  $\lambda$  <line 2> [P = f1]

x L  $\xrightarrow{12}$

PV L  $\xrightarrow{169}$

$f(r.v. f3) \rightarrow f(13) \rightarrow r.v. f4$

f3:  $\lambda$  <line 5> [P = G]

y L  $\xrightarrow{12}$

PV L  $\xrightarrow{13}$

f4:  $\lambda$  <line 4> [P = G]

x L  $\xrightarrow{13}$

PV L  $\xrightarrow{169}$